downstream of the missile nose (measured along the body) are shown in Fig. 3 for the three configurations. Survey planes were measured normal to the freestream direction, rather than to the missile axis. The spanwise dimension y and the vertical distance from the missile centerline z have been made dimensionless by the model diameter d. The location of the missile center in the crossplane is accurately plotted, although the missile cross section is not shown as elliptical. Multiple vortices may be expected this far back in the flowfield for the 13-caliber missile. The view in the figure is looking downstream.

In each case, the right vortex is slightly below the survey plane, close to the missile. The left nose-generated vortex has moved up and out of the survey plane for the body-only and $\times$ configurations. For these configurations, the vectors indicate the formation of a second left vortex below the survey grid, and close in to the centerline. In contrast, the + configuration shows the left vortex to be clearly within the survey grid, centered at $y/d = -1.0$ and $z/d = 2.8$, with no indication of a second left vortex. The two winged cases show strong inflow toward the missile body. Interestingly, the general crossplane vector field of the $\times$ case is more similar to that of the body-only than to that of the + case. Although the lee flowfields are quite different for the two wing-body configurations, the orientation does not significantly influence the effects of the forebody-generated vortices in producing large yawing moments.

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Nuclear Explosive Propelled Interceptor for Deflecting Objects on Collision Course with Earth

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Nomenclature

A_p = projected area of pusher plate, cm^2
E = energy of impact, erg
g = Earth gravitational constant, cm · s^{-2}
I_{sp} = specific impulse, s
M_a = mass of assailant, gm
M_e = mass of crater ejecta, gm
M_f = final mass of interceptor, gm
M_i = initial mass of interceptor or mass in orbit, gm
m_b = mass of nuclear explosive, gm
P = impulsive pressure at pusher plate, dyne · cm^{-2}
Q = ln (M_i/M_f) (dimensionless)
$\Re_i$ = range when assailant is intercepted, cm
$\Re_l$ = range when interceptor is launched, cm
r = distance from nuclear explosive to pusher plate, cm
t_0 = time when first debris arrives at pusher plate, s
V = interceptor velocity, cm · s^{-1}
v = closing speed of assailant, cm · s^{-1}
$v_\perp$ = assailant transverse velocity component, cm · s^{-1}
Y = energy yield of nuclear explosive, erg
α = crater constant, gm$^{(1-\beta)/2}$ · cm$^{-\beta}$ · s$^{\beta}$
β = crater exponent, dimensionless
Δt = time elapsed from launch to intercept, s
ΔV = velocity imparted to interceptor by a single nuclear explosion, cm · s^{-1}
δ = energy fraction, dimensionless
ϵ = assailant deflection distance, cm

Received March 6, 1992; revision received Sept. 17, 1993; accepted for publication Oct. 2, 1993.

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Introduction

IN 1992, the House Committee on Science, Space, and Technology mandated a study on the deflection of large space objects that might collide with the Earth.[1] The mandate reflects a heightened concern about the hazards of comets and asteroids ranging in diameter from ~100 m, such as the Tunguska object,[2] to several kilometers, such as the Cretaceous-Tertiary Impactor.[3] That our planet is in a continual state of cosmic bombardment has spurred several studies in the past.[4–6]

In a terminal defense scenario, it is generally accepted that a missile must deliver a nuclear explosive to the assailant object in order to deflect it.[7] Preparing such a missile has obvious arms-control implications. This paper shows that assailants could be deflected using an extremely high-specific-impulse interceptor without an explosive warhead. However, nuclear-explosive propulsion will be required to obtain such specific impulse.

Interceptor Flight and Assailant Deflection

Assuming it is launched from space, the final velocity of an interceptor missile relative to the Earth is given by the rocket equation

$$V = gI_{sp} \ln(M_i/M_f) \tag{1}$$

where $g \simeq 980$ cm · s^{-2}. In general, the time required to reach this relative velocity will be short compared to the total flight time. The time elapsed from launch to intercept is $\Delta t = \Re_l/(v + V)$. So the range at which the assailant is intercepted will be $\Re_i = \Re_l [1 - v/(v + V)]$. If the impact gives $v_\perp$, then the assailant will miss its target point by a distance

$$\epsilon = \Re_l \frac{v_\perp}{v} \left(\frac{V}{v + V} \right) \tag{2}$$

where the effect of the Earth's gravitational field is neglected. To obtain the transverse velocity component, we would use the kinetic energy of the interceptor to blast a crater on the side of the assailant. The momentum of the ejecta would be balanced by the transverse momentum imparted to the assailant. From Glasstone's empirical fits,[8] the mass of material ejected by a large explosion

$$M_e = \alpha^2 E^\beta \tag{3}$$

where α and β depend on the depth of the explosion, the material composition, and myriad other parameters. The composition of the assailant could be nickel–iron, stony nickel–iron, stone, chondrite, ice, or dirty snow.

The kinetic energy available when the interceptor collides with the assailant is

$$E = M_f(V + v)^2/2 \tag{4}$$

Only a fraction of the interceptor's kinetic energy is converted to kinetic energy of the ejected or "blow-off" material. Let this fraction be equal to $\delta^2/2$, a definition that simplifies the algebra. Then the transverse velocity imparted to the assailant is

$$\begin{aligned} v_\perp &= \delta \frac{\sqrt{M_e E}}{M_a} = \frac{\delta}{M_a}\sqrt{\frac{M_e M_f (V+v)^2}{2}} \\ &= \frac{\alpha\delta}{M_a}\left[\frac{M_f(V+v)^2}{2}\right]^{(\beta+1)/2} \end{aligned} \tag{5}$$

We can combine Eqs. (2) and (5) to obtain

$$\epsilon = \alpha\delta\mathfrak{R}_I \frac{V(V+v)^\beta}{M_a v}\left(\frac{M_f}{2}\right)^{(\beta+1)/2} \tag{6}$$

Equation (6) reveals the importance of V, which is proportional to I_{sp}. If $V \ll v$, the deflection is proportional to V, and if $V \gg v$, the deflection is proportional to $V^{\beta+1} \sim V^2$.

The energy on impact is proportional to the final mass of the interceptor and the square of its relative velocity, as given in Eq. (4). The smaller its final mass, the higher its relative velocity, so there is some optimum mass ratio that produces the greatest deflection for a given initial mass. This would be the optimal interceptor design.

Substituting Eq. (1) into Eq. (6), setting $d\epsilon/d(M_i/M_f) = 0$, we find the mass ratio that produces the largest value of ϵ

$$\frac{M_i}{M_f} = e^Q \tag{7}$$

where

$$Q = 1 - \frac{v}{2gI_{sp}} + \sqrt{1 + \frac{1-\beta}{1+\beta}\frac{v}{gI_{sp}} + \left(\frac{v}{2gI_{sp}}\right)^2} \tag{8}$$

We note that this optimal mass ratio depends only on the velocity of the assailant relative to the Earth and the interceptor's specific impulse. Combining Eqs. (6), (7), and (8), we find the initial mass required to achieve an impact dislocation ϵ

$$M_i = 2e^Q\left[\frac{M_a v\epsilon}{\alpha\delta\mathfrak{R}_I g I_{sp} Q}\left(\frac{1}{v + gI_{sp}Q}\right)^\beta\right]^{2/(\beta+1)} \tag{9}$$

We can appreciate the difficulties of a chemically propelled interceptor by considering an example. Assume we build an interceptor with a specific impulse of 500 s, which we launch when the assailant is at a range of 1/10 a.u. = 1.5×10^7 km, in an attempt to produce a deflection of 10^4 km, which would ensure a clear miss of our planet. The assailant is a typical chondritic asteroid, 100 m in radius with closing speed of 25 km $\cdot$ s^{-1} and density of 3 gm $\cdot$ cm^{-3}. If spherical, such an object would weigh 1.26×10^{13} gm and have an impact yield of a thousand megatons of TNT. Reasonable cratering parameters for the chondrite are $\alpha = 2 \times 10^{-4}$ gm$^{(1-\beta)/2}$ $\cdot$ cm$^{-\beta}$ $\cdot$ s$^\beta$ and $\beta = 0.9$, and a defensible estimate of the energy coupling[9] is 60%, about half of which is converted to kinetic energy of the blow-off, corresponding to $\delta \simeq 0.775$.

Using these numbers in Eq. (9), the initial mass of the chemically propelled interceptor would have to be 6,200 tons, which clearly is impractical.

Nuclear-Explosive Propelled Interceptor

Nuclear explosive propulsion was first considered in the late 1950s and early 1960s under the ORION[10] program. To get a feel for the tremendous potential of nuclear-explosive propulsion we need an estimate of the specific impulse, obtained by calculating the pressure impulse imparted by a bomb exploded in a vacuum. It can be shown[11] that the approximate pressure applied to a pusher plate a distance r from the nuclear detonation is

$$P = \frac{1}{8\pi}\sqrt{\frac{2m_b^5}{5Y^3}}\frac{r^2}{t^5}\left(1 - \frac{m_b r^2}{10Yt^2}\right).$$

The thrust is zero until the first debris arrives at the pusher plate, which occurs at $t_0 = r\sqrt{m_b/10Y}$. The velocity imparted by a single explosion is

$$\Delta V = \frac{A_p}{m}\int_{t_0}^{\infty} P\, dt = \frac{25A_p}{24m\pi r^2}\sqrt{\frac{2m_b Y}{5}}$$

If we use n bombs, the final velocity of the interceptor is

$$V = \frac{25}{24}\frac{A_p}{\pi r^2}\sqrt{\frac{2m_b Y}{5}}\sum_{j=0}^{n}\left(\frac{1}{M_i - jm_b}\right) \simeq \frac{25}{24}\frac{A_p}{\pi r^2}\sqrt{\frac{2Y}{5m_b}}\ln\frac{M_i}{M_f}$$

where the right-hand side is the limit for a very large number of bombs ($n \to \infty$) and $gM_f = g(M_i - nm_b)$ is the "dry weight" of the interceptor. By analogy with Eq. (1), we have

$$I_{sp} = \frac{25}{24g}\frac{A_p}{\pi r^2}\sqrt{\frac{2Y}{5m_b}}.$$

If the pusher plate subtends a solid angle of 2π, a bomb weighing 25 kg with a yield of 2.5 kilotons $\simeq 10^{20}$ ergs would produce a specific impulse $I_{sp} \simeq 4.25 \times 10^4$ s, assuming most of the energy goes into debris motion. This specific impulse is enormous even compared to other forms of nuclear propulsion, such as Rover-NERVA, which could approach 1,000, s or gas-core reactors, which might approach 2,000 s.

An important feature making nuclear-explosive propulsion particularly appropriate to a high-performance interceptor is that the components can be made to withstand extremely high accelerations. Arming, fusing, and firing systems of artillery shells are routinely designed to tolerate $\sim 10^3$ g. An interceptor with similarly sturdy components can attain high velocities with only a few explosives and small shock absorbers, or no shock absorbers at all. When more delicate cargo was vital to the mission, it was necessary to either use an enormous number of explosives[12] or enormously weighty shock absorbers.[13]

If we assume the specific impulse given above, then from Eq. (9) for a typical asteroid, the interceptor need weigh a mere 3 1/3 tons to deflect the same chondritic asteroid as described above. From Eqs. (7) and (8) we obtain $M_i/M_f = 7.19$, so the interceptor consists of about 2,884 kg of nuclear explosives and about 466 kg of inert components: pusher plate, shock absorbers, missile body, guidance, etc. The 115 nuclear explosives would have a total yield of 288 kilotons. From Eq. (1), the interceptor velocity at impact is 821 km $\cdot$ sec^{-1}, and from Eq. (4), the energy of the impact is 1.61×10^{21} erg = 38 kilotons. From Eq. (2), the range at intercept is 1.46×10^7 km. The time from launch to intercept is about 5 h. Thus, there would be ample time to launch a second interceptor, should

Table 1 Comparison of chemically and nuclear-explosive-propelled interceptors[a]

	Chemical	Nuclear
Specific impulse	500 s	42,500 s
Mass ratio, M_i/M_F	3.44	7.19
Initial mass	6,202 tons	3.35 tons
Final mass	1,803 tons	466 kg
Rocket velocity	6.06 km · s^{-1}	821 km · s^{-1}
Intercept range	293 Mm	14.6 Gm
Intercept time	5.6 days	5 h[c]
Collision energy	8.67×10^{21} erg[b]	1.61×10^{20} erg
	207 kT H.E.	38 kT H.E.
Blow-off mass	2.21 MT	486 kT
Fraction ejected, M_e/M_a	17.6%	3.86%

[a]Assumes 100-m radius asteroid with density 3 g · cm^{-3}; mass M_a = 12.6 MT; velocity v = 25 km · s^{-1}. Crater parameters: $\beta = 0.9$ and $\alpha = 2 \times 10^{-4}$ gm$^{(1-\beta)/2}$ · cm$^{-\beta}$ · s$^{\beta}$. Thirty percent energy to blow-off ($\delta = 0.775$).
[b]Collision will probably cause asteroid to break up.
[c]You can shoot more than once.

the first malfunction. From Eq. (3), the mass of the ejecta is about 4.86×10^{11} gm or about 3.86% of the asteroid's mass. The interceptors would most likely be stationed at an Earth–moon Lagrange point, so the fission products from the nuclear-explosive propellant would be dispersed well outside the Earth's magnetosphere.

Table 1 compares the chemically and nuclear-explosive-propelled interceptors if launched when the asteroid is 1/10 a.u. from Earth. If detected at that range, about a week would remain before the assailant collides with the Earth. The chemically propelled interceptor would have only one chance.

Conclusions

The effectiveness of using nuclear-explosive-propelled interceptors derives mainly from the fact that the interception and deflection occur farther from the Earth. The numbers used for the example are somewhat arbitrary, but the essential conclusions hold over a broad range of assumptions. The interceptor deflects the incoming object by kinetic energy alone, which might make it more politically acceptable than one bearing a nuclear warhead.

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Modeling the Short-Term Evolution of Orbital Debris Clouds in Circular Orbits

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Nomenclature

a = semimajor axis of orbit, km
c = maximum cross-track dimension of cloud, km
d = maximum radial dimension of cloud, km
E = orbital energy per unit mass, km^2/s^2
e = eccentricity of fragment orbit
i = inclination of orbit, radians
k_m = series of Fourier constants
n = mean motion of orbit, rev/s
r = radial vector, km
V = volume, km^3
v = inertial velocity, km/s
Δv = maximum velocity impulse imparted to fragments, km/s
θ = true anomaly from point of breakup, radians

Subscripts

$-\Delta v$ = specific orbital energy less than parent satellite
$+\Delta v$ = specific orbital energy greater than parent satellite

Introduction

THE breakup of a satellite in orbit will result in the formation of a debris cloud. In the absence of perturbations such as atmospheric drag and gravitational anomalies, this cloud takes the form of a torus after several days. The torus has a pinch point at $\theta = 2m\pi$ ($m = 0, 1, 2, \ldots$) and a pinch wedge at $\theta = (2m + 1)\pi$, where θ is the angular displacement from the point of breakup. The torus defines the maximum extent of the cloud envelope in inertial space and is completed when the extremes of the cloud (relating to the fragments with the greatest and least orbital energies E and denoted by $\theta_{-\Delta v}$ and $\theta_{+\Delta v}$, respectively, in Fig. 1) extend 2π away from the parent satellite locus (the position in inertial space that the satellite which breaks up would have occupied had it remained intact). In this Note, we derive a new analytic model for the evolution of the debris cloud under such conditions.

Current Modeling

The Chobotov model,[1] based on the linear relative motion in a circular orbit, predicts the cloud volume following an isotropic breakup by an expression of the form:

Received Jan. 30, 1993; revision received May 14, 1993; accepted for publication May 21, 1993.

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